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# MOBILE BREAKWATER STUDY

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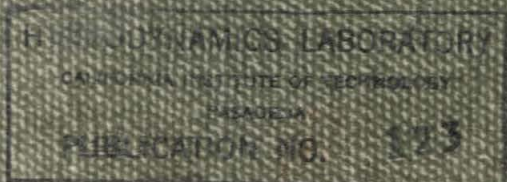
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## INTERIM REPORT

December, 1951

CALIFORNIA INSTITUTE OF TECHNOLOGY  
Hydrodynamics Laboratory, Hydraulic Structures Division  
Department of the Navy Contract NOy-12561



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Interim Report - December 1951

MOBILE BREAKWATER STUDY

Hydrodynamics Laboratories  
Hydraulic Structures Division  
of the  
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Pasadena, California

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### The Cover

The cover photographs are enlargements of frames from a 16 mm motion picture record of the performance of a three-dimensional model of a water-mass breakwater protecting a simulated cove on an exposed coast.

The disturbance in the lee of the breakwater is due almost entirely to waves diffracted into this region after passing through the opening, the breakwater itself passing very little wave energy.

The complete motion picture record is on file in the Bureau.



## I. INTRODUCTION

The Interim Report of October, 1951, presented a survey of the surface barrier studies which had been conducted by the Hydraulic Structures Laboratory up to that date. The performance curves of a number of types of mobile breakwaters were examined, and the decision reached that the most satisfactory one, all factors considered, was the three-bulkhead structure. After investigating the effect of such parameters as freeboard height, bottom clearance, and bulkhead spacing on the overall behavior of the barrier, a scale model of a hypothetical prototype pontoon assembly was constructed which incorporated what appeared to be the most effective values of these parameters. Performance data of this so-called optimum breakwater was given in the October report.

The present report continues where the previous one left off, with a more intensive consideration of certain features of barrier performance which have been but vaguely understood. Specifically, the values of the coefficient of transmission, defined as the ratio of transmitted wave height to incident wave height, were determined under various controlled wave conditions for the following bodies:

- (1) Fixed single bulkheads of different bottom clearances.
- (2) Fixed three-bulkhead barrier.
- (3) Floating three-bulkhead barrier with fixed baffle extending upward from the bottom.
- (4) Floating barrier with weighted mooring lines.

- (5) Floating barrier with increased virtual mass on the end bulkheads.
- (6) Floating barrier with hydrofoil added forward of first bulkhead.

In general, these conditions imposed upon the barrier were artificial, and impractical, as far as direct application to a prototype structure is concerned. Some information was gained, however, by separating to a limited degree the various factors which influence energy transmission by the floating breakwater.

Unless specified otherwise, the floating barrier referred to in this report is the optimum structure with three bulkheads spaced at 73 and 127 feet, high freeboard, and 15-foot, 5-foot and 15-foot bottom clearances. The original performance curve of this body, obtained using soft springs in the mooring lines, is the second curve of Fig.8 in the October report, and is used frequently as a convenient basis of comparison for the data obtained in the present series of tests.

## II. PROCEDURE

The experimental technique and the method of evaluating data are identical with those employed before in similar work. Again it is convenient to deal only with prototype dimensions on the basis of the 40 : 1 scale previously adopted. The incident waves supplied the system for these tests were 800, 600, 400, 360, 320, 280, 240, 200, 160, 120, and 100 feet long and averaged about eight feet in height in the 40-foot depth of water.

In the laboratory channel it was desirable to use a wave height somewhat less than the ten-foot height on which a prototype design would be based. At the same time, it is difficult to control incident wave height to the extent that exactly the same conditions exist from one test to the next. For these reasons an additional series of runs was made for most barrier situations to determine the effect of height variation on the transmission coefficients at constant wave length.

While there was some variation with height in connection with certain of the artificial conditions, such as the fixed single bulkheads, the slope,  $\frac{dC_T}{dh_I}$ , of the transmission coefficient curves for the normal floating barrier was almost perfectly horizontal, for shallow-water waves at least, for wave heights up to ten feet. It is well to keep in mind, however, that this fact may not hold for waves much higher than ten feet, say of the order of fifteen

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to twenty feet. If a prototype floating barrier is considered for protection against such ultra-high waves, therefore, additional experimental investigation should first be performed to determine the validity under those conditions of coefficients determined for the ten-foot heights.

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## III. FIXED SINGLE BULKHEADS

In the October report was pointed out the suspicion that energy flow beneath the floating barrier was the major factor determining the height of the transmitted wave. To aid in understanding floating barrier performance, a sequence of fixed single vertical bulkheads of different bottom clearances and of high and zero freeboard were tested one at a time. The height of the transmitted wave is, in the high freeboard cases, a direct measure of the energy passed only through the clearance area of a bulkhead fixed in space.

Each of these bulkheads was constructed of a piece of plywood which spanned the full 160-foot width of the channel, and thus maintained the two-dimensional flow situation to which the Laboratory has adhered in all small-channel studies. The plates were reinforced for rigidity and firmly clamped to the walls of the channel. The clearance distances, from channel bottom to the bottom of the plates, were about seven, thirteen and twenty feet for the high-freeboard cases, and for the zero-freeboard bulkheads, where the tops were just awash, bottom clearances of about seven and 24 feet were employed.

The performance curves of these bulkheads are given in Fig.1. A direct comparison of Figs.1(a) and 1(c) shows that the plates with tops just awash and with seven-foot and 24-foot bottom clearances transmitted more wave energy for all wave periods than the high-



freeboard bulkheads having corresponding bottom clearances. The dissipative effect of the water washing over the top was apparently insufficient to balance the increased amount of energy so allowed to pass. This result is in agreement with that observed by Johnson, Fuchs and Morison<sup>(1)</sup>: a wave does not break over a submerged barrier, especially a narrow one, unless the wave is already in a condition of instability.

Data given in the October report, on the other hand, indicated that there was little significant difference between the protection provided by a floating barrier with all freeboards high and by one having zero freeboard on the first two bulkheads. This fact, seemingly anomalous in the light of the above discussion, may be explained by the shielding action provided by the high freeboard of the third bulkhead.

While the distribution of points in Fig.1(c) may be entirely due to experimental error, it is noted that there is present at least the hint of cyclicity with peaks at wave lengths which are roughly multiples of three feet. If the curves are actually supposed to be of periodic nature, a plausible explanation lies in the possibility that for those wave lengths the "splashover" and the energy flow under the board may be more nearly in phase.

A consideration of curves (a) and (b) of Fig.1 in the light of certain well-known facts about wave behavior suggests something of the origin of the transmitted energy, which passes under the bulk-

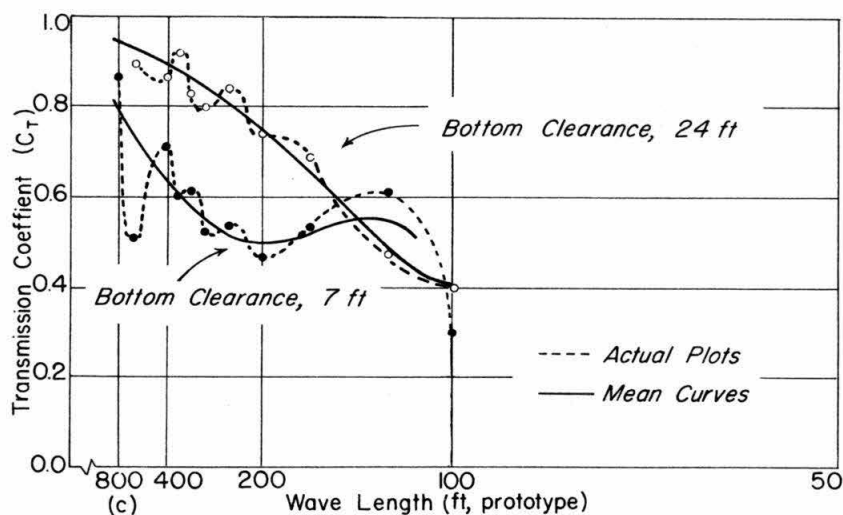
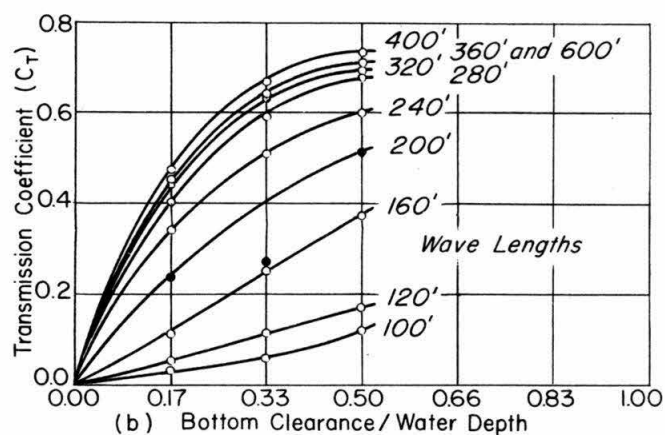
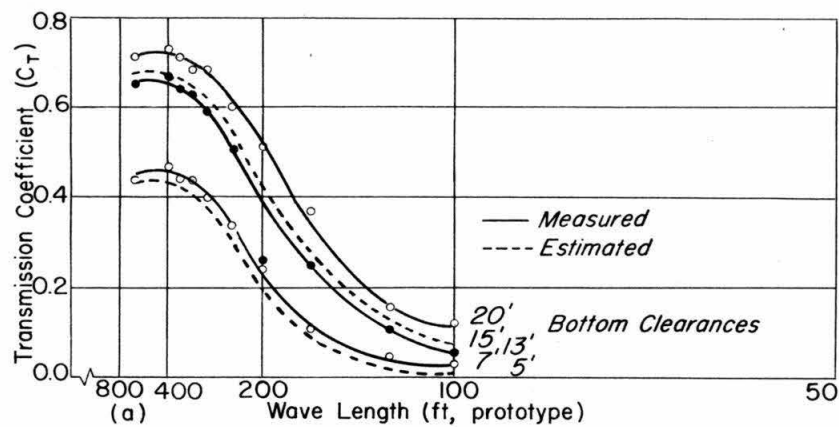


Fig. 1 - Energy Transmission Past Single, Fixed Vertical Bulkheads  
 (a) Transmission coefficient vs. wave length. (High freeboard)  
 (b) Transmission coefficient vs. bottom clearance. (High freeboard)  
 (c) Transmission coefficient vs. wave length. (Zero freeboard)

head as kinetic energy. This energy may previously have existed in one of two forms, either as a difference of potential energy between the two sides of the bulkhead, or as kinetic energy stored in the motion of the fluid particles.

In a shallow water wave the distribution of the horizontal components of particle motion is for all practical purposes uniform from top to bottom. If all the transmitted energy was provided by the kinetic energy of the wave, therefore, one would expect the ratio of transmitted to incident energy to be approximately equal to the ratio of bottom clearance to water depth. That such a situation approximately exists may be seen by considering, for example, a wave of 600-foot length, which is very close to the shallow water range in the 40-foot water depth. Corresponding to clearance-to-depth ratios of .17, .33 and .50, the expected transmission coefficients would be .41, .58 and .71. The actual experimental coefficients of .44, .63, and .69 are sufficiently close to these figures to indicate a close relationship between energy transmission and the kinetic energy of the incident wave, and the relative insignificance of the potential difference across the board.

Since the transmission coefficients for deep water waves are very small, it is obvious that differences of potential energy again do not produce a great amount of transmission. Neither, however, does the kinetic energy of the wave. This latter fact is to be expected from the variation of particle velocities as a hyperbolic

function of depth<sup>(2)</sup>, with most of the kinetic energy being concentrated very near the surface. That there is some influence present other than kinetic energy is evidenced by the fact that the curves for the long-period waves in Fig.1(b) are concave downward instead of being slightly concave upward. The minor part played by the potential differences for both extremes of wave types, however, suggests that they may be unimportant for all waves. As a matter of fact, qualitative observations of the single bulkhead in the model indicate that the net difference of potential during one wave period is very slight.



#### IV. FIXED THREE-BULKHEAD BARRIER, AND FLOATING BARRIER WITH FIXED BAFFLE

A direct study has been made which tends to substantiate the belief that for long waves fluid flow is the principle vehicle of energy transfer past the floating barrier, rather than the motion of the barrier itself. The study was conducted in two steps. First, the three-bulkhead barrier was clamped firmly to the channel walls to eliminate barrier motion, and energy which could travel only under the bulkheads was measured in terms of transmitted wave heights. Next the barrier was set afloat again and fixed vertical baffles extending upward from the bottom were added in such a manner that the top of the baffle was higher than the bottom of the middle bulkhead, and energy transmission again measured.

The first of these two steps reveals the approximate relative magnitudes of transmission due to barrier motion and to passage through the clearance area. Fig.2(a) shows the transmission coefficients of the fixed barrier and compares them directly with the coefficients for the same barrier floating. It is noted immediately that the curve for the floating barrier exhibits peaks in the vicinity of 100-foot and 320-foot wave lengths, which are considerably higher than the transmission by the fixed barrier for the same wave lengths. This apparent amplification by the barrier, as the last report pointed out, was not necessarily a result of more energy

being transmitted by the motion of the barrier itself. It is rather a combination of this factor and a more favorable phase relationship between barrier and water motion which permits an accelerated passage of energy for those wave periods.

A definite conclusion may, however, be drawn from a comparison of the two curves. If the range of wave lengths to which the floating barrier is likely to be exposed is from 200 feet up, it would not be desirable, even if possible, to fix the barrier in space in view of the fact that a peak and a valley are approximately averaged out by the fixed barrier. Further, if waves of a minimum of 400-foot length are expected, considerably better results are produced by the floating than by the fixed barrier.

The curve for the fixed barrier, meanwhile, is asymptotic to zero for very short wave lengths, as one would expect from the discussion of Sec.III. It may, therefore, be observed that whereas barrier motion bears a high degree of significance over certain ranges of the spectrum of waves whose lengths are greater than that of the barrier, for shorter waves the transmission appears to be determined by barrier motion.

An interesting fact is revealed by the third curve in Fig.2(a). This curve is a plot of the products of transmission coefficients estimated for fixed bulkheads having a 15-foot and a five-foot clearance, Fig.1(a). This curve is seen to lie very close to the transmission curve of the fixed barrier for most of its length. It

is not to be construed that the bottom clearance of the third bulkhead is insignificant; on the contrary, previous tests reveal that the opposite is true. Without further experimentation on barriers of other bulkhead spacing and bottom clearance, it can only be assumed that the result is coincidental in this particular case.

The question naturally arises as to what becomes of the energy which passes the first bulkhead but not the second. In addition to energy lost in turbulence, of course, some is reflected back out to sea from the second barrier. It is known that the potential energy of the water mass between the first two barriers is increased somewhat by part of the superfluous energy. This stored-up energy then bleeds back, part to seaward and part toward the beach. It is to be expected that the part which passes toward shore produces a wave of its own which is slightly delayed in phase with respect to the original pulse of energy, thus producing some cancellation.

The second step in this study, that of attempting to eliminate energy passage under the barrier, while measuring that passed, or permitted to pass, by the barrier motion, proved to be an unfair test and hence inconclusive. The introduction of fixed baffles extending upward from the bottom modified the barrier motion which existed before the baffles were installed, and perhaps introduced new reflections between baffle and bulkhead. At any rate, it is not surprising that the sum of the energies indicated by the curves for a fixed barrier and for a barrier with a baffle does not equal that

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for the original floating barrier as one would expect in a conservative system. The only seemingly valid deduction which may be drawn from the curves of Fig.2(b) lies in the closeness of the curves for the 12-foot and for the 30-foot baffles. The smaller one closed the energy passage route as effectively as did the large one.

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## V. WEIGHTED MOORING LINES

In accordance with part of the future program envisaged by the October report, a mooring system was devised to simulate the expected prototype design. A bar of babbit weighing 3-3/4 pounds (the equivalent of 240,000 pounds in prototype) was attached to each of two mooring lines forward and two aft of the barrier. In one test the line from the bottom of the end bulkheads to the weights extended downward and outward at 45°, and in the other the line was vertical. In each case the lines were run horizontally along the bottom from the weights to the anchor points, and in still water the weights just rested on bottom.

It was expected that the vertical weighting would be more effective in attenuating barrier motion for the "rocking" range of waves, those up to 400 feet which give the barrier primarily a rocking motion, since part of the energy of barrier motion would be absorbed in lifting the weights vertically. Waves of 600-foot and 800-foot length normally impart to the barrier primarily an oscillating translatory motion. The 45° weight suspension was to provide better resistance to motion of this type. Fig. 2(c) offers a comparison of barriers using the two methods of weighting with each other, and with the so-called optimum barrier. It is noted that the curves for the barriers with weighted lines are almost perfectly parallel, and that over the whole spectrum of waves the curve for the vertical suspension provides a slight but

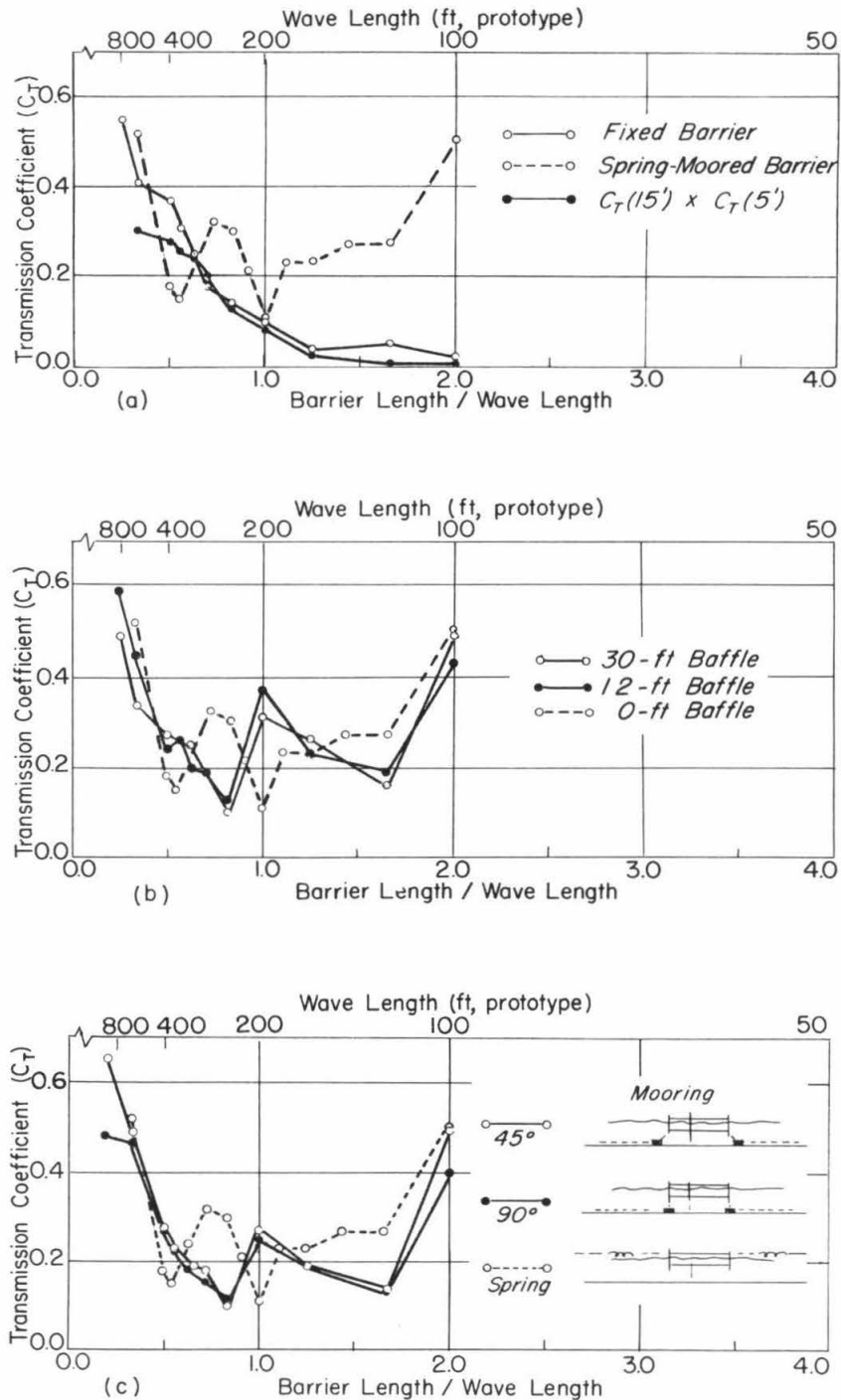


Fig. 2 - Transmission Coefficient vs. Wave Length for Three-Bulkhead Barriers

- (a) Fixed barrier
- (b) Floating barrier with fixed vertical baffles
- (c) Floating barrier with weighted mooring lines

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consistent increase in protection. The breakwater moored in this manner appears to possess generally lower transmission coefficients than those moored in the original fashion. A point of interest is the shift in the location of the resonance peak occasioned by the change from spring-mooring to weight-mooring.

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VI. FLOATING BARRIER WITH HYDROFOIL,  
AND WITH SUSPENDED FLAT PLATES

It has been mentioned frequently that one method of increasing the sheltering is to alter the phase of barrier motion with respect to that of the water. Such a change has been attempted in the Laboratory by means of a hydrofoil consisting of a horizontal flat plate of 20 feet breadth in direction of wave propagation mounted on two rigid cantilevers ahead of the barrier, and floating on the surface of the water. The wave would of course reach the plate a finite length of time before it arrived at the breakwater proper. Operating through a 200-foot moment arm, the water action on the hydrofoil produced a barrier motion which was shifted in phase from the original motion by an amount which was a function of  $l/\lambda$ . The drawback of this experiment, even as an academic study, is that for effective results in diminishing energy transmission the amount of phase change must be closely regulated. For conditions of changeable wave period, use of such a device would entail a frequent change in length of the cantilever.

A certain amount of increase in the virtual mass of the end bulkheads was achieved by attaching flat plates to the under side by tension members in such a manner that when the water was still the plate just rested on bottom. Upward motion of a bulkhead could then be expected only by moving the plate plus a mass of water, and the plate returned to its original position by gravity action.



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The steel plate used on each end was 20 feet by 110 feet by  $2\frac{1}{2}$  inches, and weighed 211,200 pounds. The desired result was achieved, in that the barrier motion was visibly reduced to a certain extent, but as Sec.IV pointed out, even a completely fixed barrier may not necessarily provide better protection than one which is free for all wave periods. The performance curves of the hydrofoiled barrier and of the barrier with suspended flat plates revealed no information of considerable significance, and for that reason are not reproduced here.

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## VII. CONCLUSIONS AND COMMENTS

The most important information gained from this series of experiments may be summarized in a number of conclusions.

(1) Energy passed by a fixed vertical bulkhead of a given bottom clearance is greater if the top is awash than if there is sufficient freeboard to prevent overtopping by the waves.

(2) The energy transmission past a high-freeboard bulkhead is predominantly a function of the kinetic energy within the incident wave, for long-period conditions. For both long- and short-period waves the importance of the potential difference across the board is but minor.

(3) The original floating barrier, moored with horizontal lines containing soft springs, provided better protection from waves longer than 400 feet than the same barrier rigidly fixed. It provided at least as good average protection for waves longer than 200 feet. Energy transmission by waves shorter than 200 feet is almost entirely a function of barrier motion.

(4) Employment of a mooring system in which weights absorb part of the energy of barrier motion is feasible for use with the compartmented barriers. The optimum magnitude and arrangements of weights is not known.

(5) It appears at present that there are but two methods of obtaining better performance from the floating barrier of this type than has been observed so far:

(a) Shifting the phase of the barrier motion with respect to that of the water in such a way that the barrier tends to reflect wave energy instead of "gulping" it. While this result may be achieved by some artificial method such as the hydrofoil described in Sec. VI, practically it could be done only by some modification of the barrier itself.

(b) Combination of high positive bulkhead buoyancy and heavy vertical mooring. Fewer of the pontoons would be flooded than in the normal floating barrier, but the structure would be 'pulled down' into the water by heavy mooring to the desired maximum clearance. By such means, the vertical component of barrier motion could be virtually eliminated, thus permitting the bottom clearances to be of equal magnitude and removing the danger of collision with the bottom.

(6) It is possible to attenuate barrier motion somewhat by increasing the effective mass of the end bulkheads with horizontal flat plates. The improvement in performance so achieved, however, is slight.

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(7) If design of a prototype is anticipated for use against waves higher than ten feet, it may be desirable to perform additional experimental study to verify the validity for higher waves of transmission coefficients which have been obtained for ten-foot waves.

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## VIII. FUTURE PROGRAM

It is believed that further work on the mobile breakwater project can most efficiently be done after personnel familiar with design of prototype structures have had sufficient opportunity to study submitted data and to offer suggestions. For that reason, additional studies of the floating bulkheaded barriers will be temporarily suspended in favor of other urgent work.

When this experimental study is resumed, however, there are certain problems which may profitably be considered, among them the following:

- (1) Measurement of forces exerted on mooring lines.
- (2) Further investigation of weight arrangement and location in mooring lines.
- (3) Further experimentation with barrier modifications which may alter its motion.
- (4) Test of a high buoyancy - heavily moored barrier as discussed in the previous section.



## A p p e n d i x

### THEORY OF MOORED FLOATING MASS BREAKWATER PERFORMANCE

In a previous report<sup>(3)</sup>, equations describing the transmission and reflection characteristics of a freely-floating mass breakwater were derived by means of momentum considerations, and the results obtained were shown to be in agreement with a much more elaborate analysis by Fritz John.

It is also desirable to obtain analytic expressions for the reflection and transmission by a floating mass breakwater with elastic horizontal restraints, since such restraints will be present in any prototype in the form of a mooring system. This problem involves the solution of the dynamic equation of the system:

$$M\ddot{x} + Kx = F(\tau)$$

with the continuity conditions that the horizontal particle velocity at each vertical face of the barrier is continuous with the barrier motion itself.

Before solving this problem it will be instructive to consider the simpler case of a freely-floating body (as solved previously) by use of the equation of dynamics:

$$M\ddot{x} = F(\tau)$$

For shallow-water waves, the force on the mass may be taken as due to the pressure heads acting on the seaward and leeward faces of the barrier:

$$F = \frac{1}{2} w (d + y_1)^2 - \frac{1}{2} w (d + y_2)^2$$

$$= \frac{1}{2} w [2d (y_1 - y_2) + (y_1^2 - y_2^2)]$$

$$F = wd (y_1 - y_2) \text{ (neglecting square of small quantities)}$$

where:  $w$  = specific weight of sea water

$d$  = mean water depth

$y_1$  = water surface elevation measured from still water level at seaward face of barrier

$y_2$  = water surface elevation measured from still water level at leeward face of barrier.

$y_1$  is due to the incident and reflected wave trains, and  $y_2$  is due to the transmitted train; or:

$$y_1 = i \sin 2\pi \frac{\tau}{T} + r \sin (2\pi \frac{\tau}{T} + \alpha)$$

$$y_2 = t \sin (2\pi \frac{\tau}{T} - \beta)$$

where  $\alpha$  and  $\beta$  are arbitrary phase angles which must be determined by the continuity requirements.

Since for progressive wave trains the horizontal particle velocity ( $V$ ) is in phase with the wave height:

$$V_{\text{sea}} = K [i \sin 2\pi \frac{\tau}{T} - r \sin (2\pi \frac{\tau}{T} + \alpha)]$$

$$V_{\text{lee}} = K [t \sin (2\pi \frac{\tau}{T} - \beta)]$$

and from the continuity condition:

$$V_{\text{sea}} = V_{\text{lee}}$$

$$i \sin 2\pi \frac{\tau}{T} - r \sin 2\pi \frac{\tau}{T} \cos \alpha - r \cos 2\pi \frac{\tau}{T} \sin \alpha$$

$$- t \sin 2\pi \frac{\tau}{T} \cos \beta + t \cos 2\pi \frac{\tau}{T} \sin \beta = 0$$

$$\text{or: } (i - r \cos \alpha - t \cos \beta) \sin 2\pi \frac{\tau}{T} + (-r \sin \alpha + t \sin \beta) \cos 2\pi \frac{\tau}{T} = 0,$$

from which:

$$(i - r \cos \alpha - t \cos \beta)^2 + (-r \sin \alpha + t \sin \beta)^2 = 0$$

and finally using the conservation of energy condition:

$$i^2 = r^2 + t^2;$$

$$2i^2 - 2i (r \cos \alpha + t \cos \beta) + 2rt \cos (\alpha + \beta) = 0$$

from which, by inspection

$$\cos \alpha = \sin \beta = \frac{r}{i}$$

$$\cos \beta = \sin \alpha = \frac{t}{i}$$

$$\text{and } F = 2wdr \cos (2\pi \frac{\tau}{T} - \beta)$$

$$\dot{x} = \frac{2wdr}{M} \cos (2\pi \frac{\tau}{T} - \beta)$$

$$\dot{x} = \frac{wdrT}{Mt} \sin (2\pi \frac{\tau}{T} - \beta) + C_1$$

$$x = \frac{wdrT^2}{2Mt^2} \cos (2\pi \frac{\tau}{T} - \beta) + C_1 t + C_2$$

From continuity;  $\dot{x} = V_{\text{sea}} = V_{\text{lee}}$ ,

and  $\dot{x} = 0$  when  $2\pi \frac{\tau}{T} = \beta$ ,

or  $C_1 = 0$

Since the horizontal velocity and displacement of the waves are  $90^\circ$  out of phase:

$$x = 0 \text{ when } \frac{2\pi \tau}{T} - \beta = \frac{\pi}{2}$$

or,  $C_2 = 0$

Hence the double amplitude of horizontal motion of the barrier is  $\frac{wdrT^2}{Mt^2}$  which is also the double amplitude of horizontal particle motion of the transmitted wave;

$$\text{or} \quad \frac{wdrT^2}{M\pi^2} = \frac{tL}{\pi d}$$

$$\text{putting } M = \frac{W}{g}$$

$$T = \frac{L}{C} = \frac{L}{\sqrt{gd}}$$

$$\frac{wrL^2}{W\pi^2} = \frac{tL}{\pi d}$$

$$\frac{t}{r} = \frac{wLd}{W\pi}$$

and since

$$\frac{r}{i} = \frac{1}{\sqrt{1 + \frac{t^2}{r^2}}}; \quad \frac{t}{i} = \sqrt{1 - \left(\frac{r}{i}\right)^2} = \frac{1}{\sqrt{1 + \left(\frac{r}{t}\right)^2}}$$

$$\frac{r}{i} = \frac{1}{\sqrt{1 + \left(\frac{wLd}{\pi W}\right)^2}}$$

$$\frac{t}{i} = \frac{1}{\sqrt{1 + \left(\frac{\pi W}{wLd}\right)^2}}$$

Putting the barrier weight  $W$  equal to the weight of water between the seaward and leeward faces:  $W = wd\ell$ .

$$\text{We have: } C_t = \frac{t}{i} = \frac{1}{\sqrt{1 + \left(\frac{\pi\ell}{L}\right)^2}}$$

$$C_r = \frac{r}{i} = \frac{1}{\sqrt{1 + \left(\frac{L}{\pi\ell}\right)^2}} \quad \text{as before.}$$

Proceeding now to the problem of a moored barrier, we may make use of the previous results for force applied, since the introduction of the elastic restraints does not affect the continuity relationships;

$$Mx'' + Kx = [F(T)] = 2wrd \cos \left(2\pi \frac{T}{T} - \beta\right).$$

The solution of this well known second-order linear differential equation is the sum of the free oscillation, obtained from reduced equation:

$$M\ddot{x} + Kx = 0$$

and the forced oscillation, obtained by finding a particular integral which satisfies the complete equation. Since any small damping will eventually reduce the free oscillation to zero, but affect the forced oscillation very little, the forced oscillation as obtained from the undamped equation is a good approximation of the steady-state motion.

The equation may be rewritten by expanding the cosine term:

$$M\ddot{x} + Kx = \frac{2wdr}{i} \left( r \sin 2\pi \frac{\tau}{T} + t \cos 2\pi \frac{\tau}{T} \right)$$

and a particular integral found by the method of undetermined coefficients:

$$\text{Let } x = A \sin 2\pi \frac{\tau}{T} + B \cos 2\pi \frac{\tau}{T}$$

$$\dot{x} = \frac{2\pi A}{T} \cos 2\pi \frac{\tau}{T} - \frac{2\pi B}{T} \sin 2\pi \frac{\tau}{T}$$

$$\ddot{x} = \frac{4\pi^2 A}{T^2} \sin 2\pi \frac{\tau}{T} - \frac{4\pi^2 B}{T^2} \cos 2\pi \frac{\tau}{T}$$

$$\begin{aligned} \text{Then: } M\ddot{x} + Kx &= -\frac{4\pi^2}{T^2} M \left( A \sin 2\pi \frac{\tau}{T} + B \cos 2\pi \frac{\tau}{T} \right) \\ &\quad + K \left( A \sin 2\pi \frac{\tau}{T} + B \cos 2\pi \frac{\tau}{T} \right) \\ &= \frac{2wdr}{i} \left( r \sin 2\pi \frac{\tau}{T} + t \cos 2\pi \frac{\tau}{T} \right) \end{aligned}$$

From which by identities:

$$A \left[ K - M \left( \frac{2\pi}{T} \right)^2 \right] = \frac{2wdr^2}{i}$$

$$B \left[ K - M \left( \frac{2\pi}{T} \right)^2 \right] = \frac{2wrtg}{i}$$

and:

$$x = \frac{2wd \frac{r}{i}}{K \left[ 1 - \frac{M}{K} \left( \frac{2\pi}{T} \right)^2 \right]} \left( r \sin 2\pi \frac{\tau}{T} + t \cos 2\pi \frac{\tau}{T} \right)$$

Putting  $S = \frac{2\pi}{\sqrt{\frac{K}{M}}}$ , the natural period of the barrier-mooring system:

$$x = \frac{2 w d r}{K \left[ 1 - \left( \frac{S}{T} \right)^2 \right]} \cos \left( 2\pi \frac{\tau}{T} - \beta \right)$$

Again equating the double amplitude of barrier and transmitted-wave particle motion:

$$\frac{4 w d r}{K \left[ 1 - \left( \frac{S}{T} \right)^2 \right]} = \frac{tL}{\pi d}$$

$$\frac{t}{r} = \frac{4\pi w d^2}{LK \left[ 1 - \left( \frac{S}{T} \right)^2 \right]}$$

and

$$\frac{t}{i} = \frac{1}{\sqrt{1 + \left\{ \frac{LK \left[ 1 - \left( \frac{S}{T} \right)^2 \right]}{4\pi w d^2} \right\}^2}}$$

$$\frac{r}{i} = \frac{1}{\sqrt{1 + \left\{ \frac{4\pi w d^2}{LK \left[ 1 - \left( \frac{S}{T} \right)^2 \right]} \right\}^2}}$$

To verify these results, we may substitute  $M = \frac{W}{g}$ ,  $T^2 = \frac{L^2}{gd}$ ,  $K = 0$ ; obtaining

$$\frac{t}{i} = \frac{1}{\sqrt{1 + \left( \frac{\pi W}{wdL} \right)^2}}$$

$$\frac{r}{i} = \frac{1}{\sqrt{1 + \left( \frac{wdL}{\pi W} \right)^2}},$$

which agrees with the previously-derived result for no restraint.



The maximum force per foot of barrier length exerted by the barrier on the anchor point is equal to the spring constant  $K$  of the mooring times the half-amplitude of motion of the barrier:

$$F_{\max} = Kx = \frac{2 w d r}{\left[ 1 - \left( \frac{S}{T} \right)^2 \right]},$$

where  $S$  and  $K$  are calculated on the basis of a barrier strip of unit length.

The corresponding maximum force on a rigid barrier is:

$$F'_{\max} = 2 w d r,$$

hence:

$$\frac{F_{\max}}{F'_{\max}} = \left| \frac{1}{1 - \left( \frac{S}{T} \right)^2} \right|$$

This relationship is plotted in Fig.3a, where it is seen that it is essential that the ratio  $\frac{S}{T}$  exceed a value of 1.414.

The effectiveness of the moored barrier may be evaluated by comparing its  $C_t$  with that of a free barrier:

$$\begin{aligned} \frac{C_{t \text{ moored}}}{C_{t \text{ free}}} &= \sqrt{\frac{1 + \left( \frac{\pi \ell}{L} \right)^2}{1 + \left\{ \frac{LK \left[ 1 - \left( \frac{S}{T} \right)^2 \right]}{4\pi w d^2} \right\}^2}} \\ &= \sqrt{\frac{1 + \left( \frac{\pi \ell}{L} \right)^2}{1 + \left( \frac{\pi \ell}{L} \right)^2 \left[ \left( \frac{1}{S/T} \right)^2 - 1 \right]^2}} \end{aligned}$$

This relationship is plotted in Fig.3b, where it is seen that the

mooring system reduces transmitted wave heights only when the natural period of the barrier-mooring system is shorter (.707 or less) than the incident wave period.

The important result of this analysis may readily be seen by comparing Figs. 3a and 3b. It will be observed that any elastic mooring system which requires less anchor force than that required by a rigidly-fixed barrier will result in higher transmitted waves than a completely free barrier of the same width. Therefore, an elastic mooring system cannot be used to increase the effectiveness of a floating mass breakwater. Since some mooring is required to resist the steady force component due to unsymmetrical waves, currents, winds, etc., the mooring should be designed to produce a barrier motion period in excess of 3 or 4 times the expected wave period, and so limit the resonant amplification effect of the mooring to a small quantity.

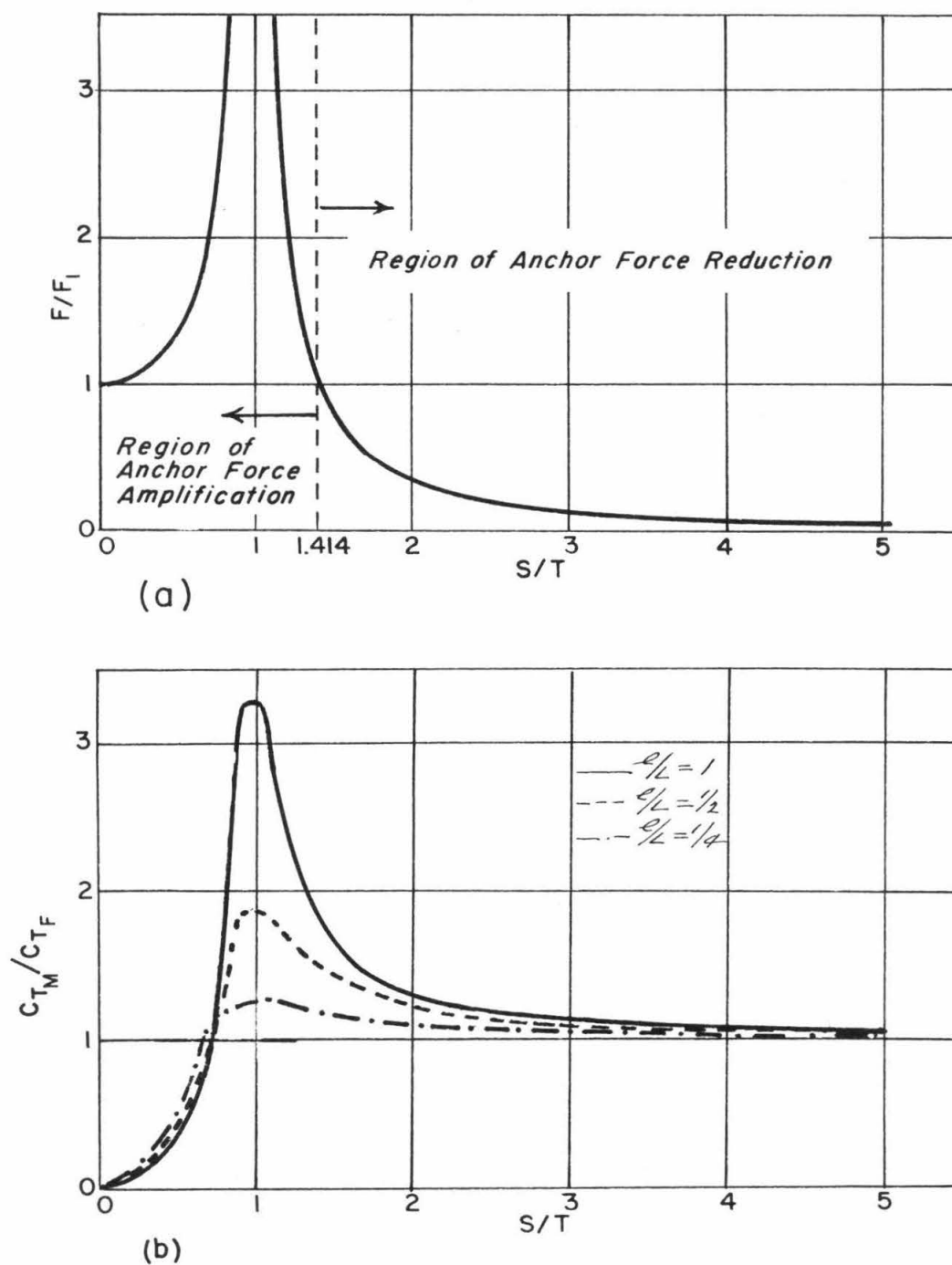


Fig. 3 - Theoretical Results for Elastically-Moored Barriers

- (a) Mooring force requirements  
 (b) Transmission characteristics

R e f e r e n c e s

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